TRANSIENTS IN THERMISTOR CIRCUITS WITH PULSED CHANGES IN HEAT-TRANSFER CONDITIONS

V. A. Palagin

Inzhenerno-Fizicheskii Zhurnal, Vol. 10, No. 2, pp. 201–211, 1966

UDC 536.244

Relations are derived for the change in thermistor temperature with pulsed changes in ambient temperature and dissipation factor for small deviations of circuit current. Three main types of input pulses are examined: square, triangular, and exponential.

In recent years pulse technology has been increasingly introduced into automatic control systems. Accordingly, it is a matter of some interest to study pulse regimes in circuits with thermistors, which are often used as components of such systems. Since the thermistor can be used to register both temperature and the parameters affecting the heat transfer coefficient, the ambient temperature and the dissipation factor were taken as input quantities. All the calculations were made for small variations of the current in the circuit under the action of an unmodulated sequence of square, triangular or exponential pulses (Fig. 1). The pulse sequence can be characterized (Fig. 1) by the pulse amplitude, repetition period, off-duty factor [1]

$$\gamma = t_i/\tau$$

and amplification factor

$$k_i = A_m/x. \tag{1}$$

The quantity x in (1) represents the value of the input quantity at a discrete moment of time. The analysis of pulse systems usually reduces to a calculation of transients, i.e., to a determination of the output quantity as a function of time z(t). The ratio of the transform of the input to the transform of the output

$$K^* = Z^*(q, \varepsilon)/X^*(q, 0) \tag{2}$$

is the transfer function of an open pulse-amplitude system. In expression (2)

$$q = p\tau \tag{3}$$

is the transformation parameter, which is introduced on going over to relative time values ($\bar{t} = t/\tau$).

We recall that any pulse-amplitude system can be reduced to a pulse system consisting of a simple pulse element, a shaping element and a continuous part.

The transfer function of a system with arbitrary pulse shape is given by the following equations [1]:

$$K^*(q, \varepsilon) = \sum_{\nu=1}^{l} c'_{\nu_0} \frac{e^{q_\nu \varepsilon}}{e^q - e^{q_\nu}} \left[e^q \int_0^\varepsilon s_0(\lambda) e^{-q_\nu \lambda} d\lambda + \right.$$

$$\left. + e^{q_\nu} \int_\varepsilon^\gamma s_0(\lambda) e^{-q_\nu \lambda} d\lambda \right] \text{ when } 0 \le \varepsilon \le \gamma, \tag{4a}$$

$$K^*(q, \varepsilon) = \sum_{\nu=1}^{l} c'_{\nu_0} \frac{e^{q_\nu \varepsilon}}{e^q - e^{q_\nu}} e^q \int_0^\gamma s_0(\lambda) e^{-q_\nu \lambda} d\lambda \text{ when } \gamma \le \varepsilon \le 1. \tag{4b}$$

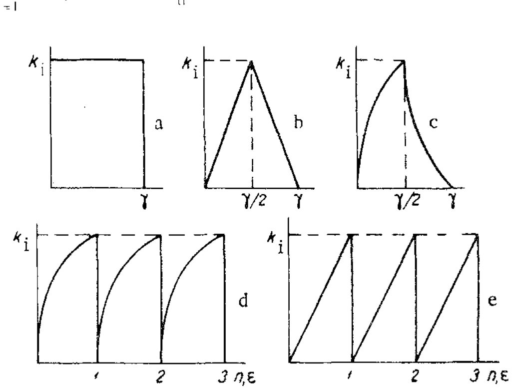

Fig. 1. Pulse shapes: a) square, b) triangular, c) exponential, d) exponential sawtooth, e) linear sawtooth.

In (4a), (4b)

$$s_0(\lambda) = s_0(\varepsilon) = \tau s(\varepsilon)/k_i \tag{5}$$

is the shape of the pulse at the pulse element output when $\tau = k_i$, and

$$c'_{\nu_0} = \frac{k_i}{\tau} \frac{P_n(q_\nu)}{Q'_n(q_\nu)}. \tag{6}$$

If we assume that the input variable is discontinuous, the transform of the corresponding lattice function [1] will be

$$X^*(q, 0) = X^*(q) = x_0 e^q/(e^q - 1). \tag{7}$$

Here x_0 is the value of the jump at the input of the simple pulse element. According to (2) and (7),

$$Z^*(q, \varepsilon) = K^*(q, \varepsilon) x_0 e^q/(e^q - 1). \tag{8}$$

If the shaping element produces a sequence of square pulses, the inverse transform of the output variable is [1]

$$z[n, \varepsilon] = x_0 \left[K^*(0, \varepsilon) - \sum_{\nu=1}^{l} c_{\nu_0} \frac{1 - e^{-q_\nu \gamma}}{1 - e^{q_\nu}} e^{q_\nu (n+1+\varepsilon)} \right], \tag{9}$$

where

$$K^*(0, \varepsilon) = c_{00} + \sum_{\nu=1}^{l} c_{\nu_0} \frac{1 - e^{q_\nu (1-\gamma)}}{1 - e^{q_\nu}} e^{q_\nu \varepsilon} \text{ for } 0 \le \varepsilon \le \gamma,$$

$$K^*(0,\ \varepsilon) = \sum_{\nu=1}^{l} c_{\nu_0} \frac{e^{q_\nu\gamma}-1}{1-e^{q_\nu}}\, e^{q_\nu(\varepsilon-\gamma)} \quad \text{for } \gamma \leqslant \varepsilon \leqslant 1,$$

and

$$c_{00} = k_i P_n(0)/Q_n(0), \quad c_{\nu_0} = \tau c'_{\nu_0}/q_\nu. \tag{10}$$

Since the literature lacks expressions $K^*(q,\ \varepsilon)$ and $Z^*(q,\ \varepsilon)$ for triangular and exponential pulse shapes (Fig. 1b, c), we obtained them on the basis of expressions (4a), (4b), and (8). To convert from the transform of a function to the original we used the relations [1]:

$$\frac{e^q}{(e^q-e^{q_\nu})(e^q-1)}\, e^q \doteqdot \frac{1}{1-e^{q_\nu}}\,[1-e^{q_\nu(n+1)}],$$

$$\frac{e^q}{(e^q-e^{q_\nu})(e^q-1)} \doteqdot \frac{1}{1-e^{q_\nu}}\,(1-e^{q_\nu n}).$$

The shape of the triangular pulses is described by the expressions

$$s_0(\varepsilon) = \begin{cases} \tau\varepsilon\cdot 2/\gamma, & 0 \leqslant \varepsilon \leqslant \gamma/2 \\ \tau(\gamma-\varepsilon)\cdot 2/\gamma, & \gamma/2 \leqslant \varepsilon \leqslant \gamma \\ 0, & \gamma \leqslant \varepsilon \leqslant 1. \end{cases} \tag{11}$$

The shape of the exponential pulses

$$s_0(\varepsilon) = \begin{cases} \tau(1-e^{-q_1\varepsilon})/a, & 0 \leqslant \varepsilon \leqslant \gamma/2, \\ \tau\left[e^{q_1(\gamma/2-\varepsilon)} + \dfrac{2}{\gamma}\left(\dfrac{\gamma}{2}-\varepsilon\right)e^{-q_1\gamma/2}\right], & \gamma/2 \leqslant \varepsilon \leqslant \gamma, \\ 0, & \gamma \leqslant \varepsilon \leqslant 1, \end{cases} \tag{12}$$

where $a = 1 - e^{-q_1\gamma/2}$.

In determining the transforms of the transfer functions for triangular and exponential pulse sequences we took into account the fact that the transform of a composite function is the sum of the transforms of the individual parts of this function with the same law [2]. Without dwelling on the details of the derivation, we present the final solutions for the inverse transforms of the output variables of the pulse systems.

For a triangular pulse sequence (11) when $0 \le \varepsilon \le \gamma/2$

$$z[n,\ \varepsilon] = \frac{2x_0}{\gamma}\sum_{\nu=1}^{l} c_{\nu 01}\{-(1+q_\nu\varepsilon) + A_1(\gamma,\ \nu)\, e^{q_\nu\varepsilon} - A_2(\gamma,\ \nu)\, e^{q_\nu\varepsilon}\, e^{q_\nu(n+1)}\}, \tag{13a}$$

when $\gamma/2 \le \varepsilon \le \gamma$

$$z[n,\ \varepsilon] = \frac{2x_0}{\gamma}\sum_{\nu=1}^{l} c_{\nu 01}\{[1-q_\nu(\gamma-\varepsilon)] + A_3(\gamma,\ \nu)\, e^{q_\nu\varepsilon} - A_2(\gamma,\ \nu)\, e^{q_\nu\varepsilon}\, e^{q_\nu(n+1)}\}, \tag{13b}$$

when $\gamma \le \varepsilon \le 1$

$$z[n,\ \varepsilon] = \frac{2x_0}{\gamma}\sum_{\nu=1}^{l} c_{\nu 01} A_2(\gamma,\ \nu)\, e^{q_\nu\varepsilon}\,[1-e^{q_\nu(n+1)}]. \tag{13c}$$

In these expressions

$$A_1(\gamma,\ \nu) = \frac{1-(2-e^{-q_\nu\gamma/2})\, e^{q_\nu(1-\gamma/2)}}{1-e^{q_\nu}},$$

$$A_2(\gamma,\ \nu) = \frac{1-(2-e^{-q_\nu\gamma/2})\, e^{-q_\nu\gamma/2}}{1-e^{q_\nu}},$$

$$A_3(\gamma,\ \nu) = \frac{1-[2-e^{q_\nu(1-\gamma/2)}]\, e^{-q_\nu\gamma/2}}{1-e^{q_\nu}},$$

and with account for expression (10)

$$c_{\nu 01} = c_{\nu 0}/q_\nu = \tau c'_{\nu 0}/q_\nu^2. \tag{14}$$

The output variables for systems whose shaping element produces pulses of exponential shape are when $0 \le \varepsilon \le \gamma/2$

$$z[n,\ \varepsilon] = x_0\sum_{\nu=1}^{l} c'_{\nu_0}\frac{\tau}{a}\left[\left(\frac{e^{-q_1\varepsilon}}{q_1+q_\nu} - \frac{1}{q_\nu}\right) + A_4(\gamma,\ \nu)\, e^{q_\nu\varepsilon} - A_5(\gamma,\ \nu)\, e^{q_\nu\varepsilon}\, e^{q_\nu(n+1)}\right], \tag{15a}$$

when $\gamma/2 \le \varepsilon \le \gamma$

$$z[n,\ \varepsilon] = x_0\sum_{\nu=1}^{l} c'_{\nu_0}\frac{\tau}{a}\left\{-a\left[\frac{e^{q_1(\gamma/2-\varepsilon)}}{q_1+q_\nu} + \frac{e^{-q_1\gamma/2}}{q_\nu}\right] + \frac{2a}{\gamma q_\nu^2}(1+q_\nu\varepsilon)\, e^{-q_1\gamma/2} + A_6(\gamma,\ \nu)\, e^{q_\nu\varepsilon} - A_5(\gamma,\ \nu)\, e^{q_\nu\varepsilon}\, e^{q_\nu(n+1)}\right\}, \tag{15b}$$

when $\gamma \le \varepsilon \le 1$

$$z[n,\ \varepsilon] = x_0\sum_{\nu=1}^{l} c'_{\nu_0}\frac{\tau}{a} A_5(\gamma,\ \nu)\,[1-e^{q_\nu(n+1)}]\, e^{q_\nu\varepsilon}. \tag{15c}$$

Here

$$A_4(\gamma,\ \nu) = \frac{1}{1-e^{q_\nu}}\,[A_7 + (A_8 - A_9 + A_{10})\, e^{q_\nu(1-\gamma/2)}],$$

$$A_5(\gamma,\ \nu) = \frac{1}{1-e^{q_\nu}}\,[A_7 + (A_8 - A_9 + A_{10})\, e^{-q_\nu\gamma/2}],$$

$$A_6(\gamma,\ \nu) = \frac{1}{1-e^{q_\nu}}\,[A_7 + A_8 e^{-q_\nu\gamma/2} - A_9 e^{q_\nu(1-\gamma/2)} + A_{11}],$$

where

$$A_7 = \left(\frac{1}{q_\nu} - \frac{1}{q_1+q_\nu}\right),$$

$$A_8 = \left(\frac{e^{-q_1\gamma/2}}{q_1+q_\nu} - \frac{1}{q_\nu}\right) + a\left(\frac{1}{q_1+q_\nu} + \frac{e^{-q_1\gamma/2}}{q_\nu}\right),$$

$$A_9 = a\left(\frac{1}{q_1+q_\nu} + \frac{1}{q_\nu}\right)e^{-(q_1+q_\nu)\gamma/2},$$

$$A_{10} = \frac{2a}{\gamma q_\nu^2}\left[(1+q_\nu\gamma)\, e^{-q_\nu\gamma/2} - \left(1+q_\nu\frac{\gamma}{2}\right)\right]e^{-q_1\gamma/2},$$

$$A_{11} = \frac{2a}{\gamma q_\nu^2}\left[(1+q_\nu\gamma)\, e^{q_\nu(1-\gamma/2)} - \left(1+q_\nu\frac{\gamma}{2}\right)\right]e^{-(q_1+q_\nu)\gamma/2}.$$

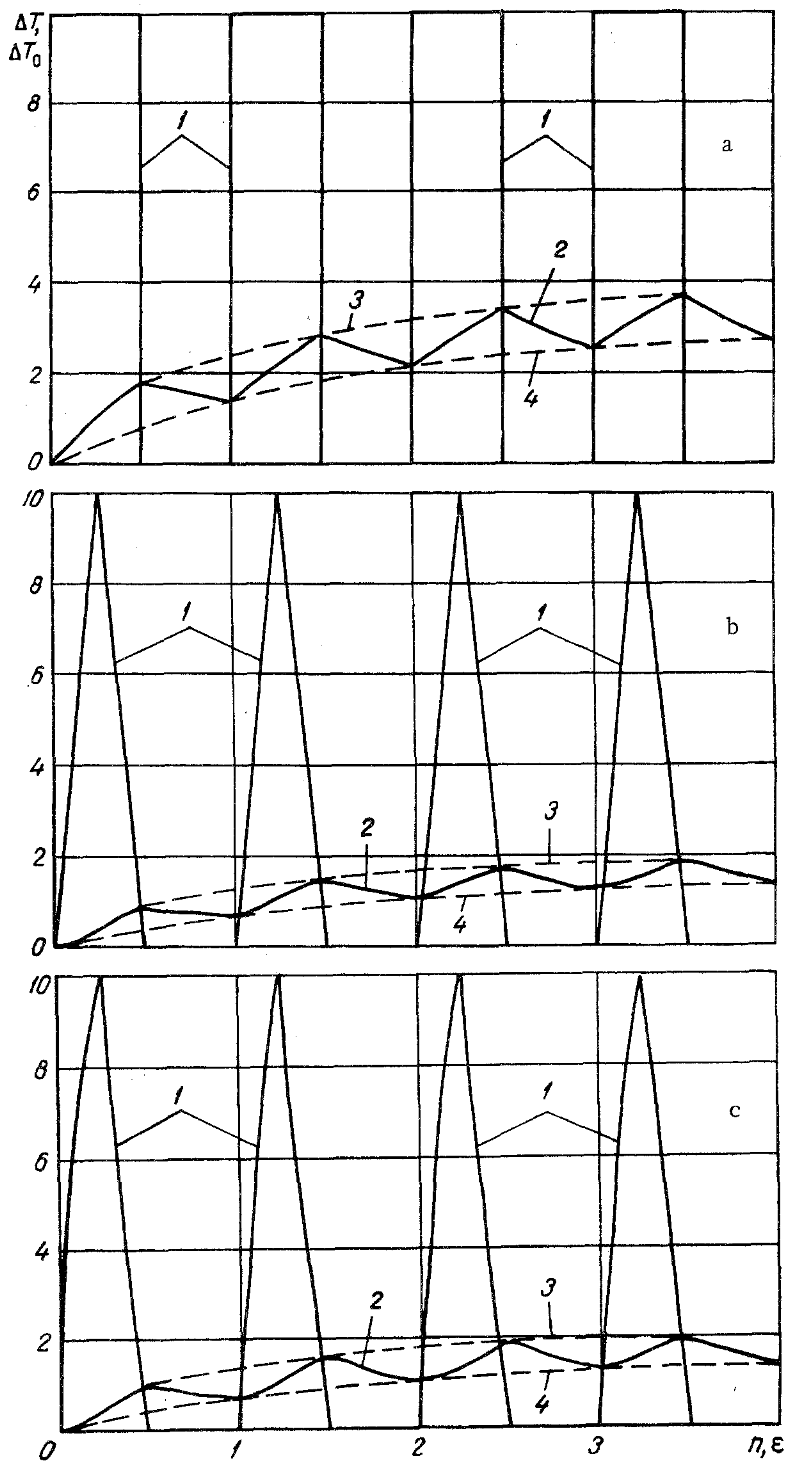

Fig. 2. Change of temperature of KMT-14 thermistor, degrees, for a sequence of square (a), triangular (b), and exponential (c) pulses of ambient temperature (thermistor and circuit parameters: R_{20} = 71 kohm, R = = 1350 ohms, D = 2.93, δ = −0.15, τ_e = 4.84 sec, amplitude of change ΔT_{0m} = 10° C, τ = 2 sec, γ = 0.5, q_1 = = 1): 1) ΔT_0; 2) ΔT; 3) ΔT_{max}; 4) ΔT_{min}.

The above expressions for the output variables are valid for linear systems. Since for small current variations a thermistor circuit can be represented in the form of a linear equivalent circuit [3–5], expressions (13) and (15) can be used to calculate transients in such circuits.

Pulsed variation of ambient temperature. Let the thermistor be connected in series with a linear resistance ($R - R_T$ circuit). We assume that U_c = const, R = const, and k = const. For small current deviations in the general case the dissipated power will be

$$\Delta P_\alpha = k(\Delta T - \Delta T_0) - (T_1 - T_0)\Delta k. \qquad (16)$$

In our case (16) assumes the form

$$\Delta P_\alpha = k(\Delta T - \Delta T_0). \qquad (17)$$

The current increment for U_c = const, with allowance for the fact that $\Delta R_T = \alpha_T R_T \Delta T$, is equal to [4, 5]

$$\Delta I = -\frac{U_c}{(R+R_T)^2}\Delta R_T = \frac{U_c \alpha_T R_T}{(R+R_T)^2}\Delta T. \qquad (18)$$

Since with variation ΔI of the current the increment ΔP_T in the power supplied to the thermistor has the form

$$\Delta P_T = 2U_T \Delta I + I^2 \Delta R_T \qquad (19a)$$

and, on the other hand,

$$\Delta P_T = \Delta P_\alpha + C_V p \Delta T, \qquad (19b)$$

using (17) and (18) we find the transfer function of the thermistor circuit:

$$\Delta T(p)/\Delta T_0(p) = K_n(p) = [(\tau_e p + 1)(1 - D\delta)]^{-1} = P_n(p)/Q_n(p).$$

The value of the transfer function expressed in terms of the transformation parameter (3) will be

$$K_n(q) = \beta_1/(q+\beta_1)(1 - D\delta) = P_n(q)/Q_n(q),$$

where $\beta_1 = \tau/\tau_e$.

Setting $Q_n(q)$ equal to zero, we get the unique root $q_{1n} = -\beta_1$. On the basis of (6), (10), and (14) we determine the coefficients:

$$c_{00} = \frac{k_i P_n(0)}{Q_n(0)} = k_i \frac{1}{(1-D\delta)}, \qquad c_{10} = \frac{k_i P_n(q_{1n})}{q_{1n} Q_n'(q_{1n})} = -k_i \frac{1}{1-D\delta}, \qquad (20)$$

$$c'_{10} = \frac{k_i}{\tau}\frac{P_n(q_{1n})}{Q_n'(q_{1n})} = \frac{k_i}{\tau}\frac{\beta_1}{1-D\delta}, \qquad (21)$$

$$c_{101} = \frac{\tau}{q_\nu^2} c'_{\nu 0} = \frac{k_i}{\beta_1}\frac{1}{1-D\delta}. \qquad (22)$$

Using (9), (20), and (21), we write the final relations for the change in thermistor temperature in the case of a sequence of square pulses, taking $k_i = i$ (i.e., $x_0 = A_m = \Delta T_{0m}$):

for $0 \le \varepsilon \le \gamma$

$$\Delta T[n,\varepsilon] = \frac{\Delta T_{0m}}{1-D\delta}\left[1 - \frac{1-e^{-\beta_1(1-\gamma)}}{1-e^{-\beta_1}} e^{-\beta_1\varepsilon} + \frac{1-e^{\beta_1\gamma}}{1-e^{-\beta_1}} e^{-\beta_1(n+1+\varepsilon)}\right], \qquad (23a)$$

for $\gamma \le \varepsilon \le 1$

$$\Delta T[n,\varepsilon] = \frac{\Delta T_{0m}}{1-D\delta}\left[\frac{1-e^{-\beta_1\gamma}}{1-e^{-\beta_1}} e^{-\beta_1(\varepsilon-\gamma)} + \frac{1-e^{\beta_1\gamma}}{1-e^{-\beta_1}} e^{-\beta_1(n+1+\varepsilon)}\right]. \qquad (23b)$$

The maximum value of ΔT will be at $\varepsilon = \gamma$, and the minimum value at $\varepsilon = 0$. Substituting the corresponding values in (23a), we get the equations of the envelopes [1]:

$$\Delta T_{\max} = \Delta T[n, \gamma]; \quad \Delta T_{\min} = \Delta T[n, 0].$$

As $n \to \infty$, the system enters a state of dynamic equilibrium. Then the maximum and minimum values of the change in thermistor temperature tend to constant values:

$$\Delta T_{\max, y} = \Delta T[\infty, \gamma]; \quad \Delta T_{\min, y} = \Delta T[\infty, 0].$$

It is easy to show [1] that a 5% difference from the steady-state values of the envelopes is observed after a number of repetition periods:

$$n \geqslant 3/\beta_1.$$

The curves in Fig. 2a are based on the above formulas.

In the case of a sequence of triangular pulses the changes in thermistor temperature according to (13) and (22) (at $k_i = 1$) are equal to

when $0 \le \varepsilon \le \gamma/2$

$$\Delta T[n,\varepsilon] = \frac{2\Delta T_{0m}}{\beta_1\gamma(1-D\delta)}[(\beta_1\varepsilon - 1) + A_1(\gamma,\nu)e^{-\beta_1\varepsilon} - A_2(\gamma,\nu)e^{-\beta_1\varepsilon}e^{-\beta_1(n+1)}], \qquad (24a)$$

when $\gamma/2 \le \varepsilon \le \gamma$

$$\Delta T[n,\varepsilon] = \frac{2\Delta T_{0m}}{\beta_1\gamma(1-D\delta)}[1 + \beta_1(\gamma-\varepsilon) + A_3(\gamma,\nu)e^{-\beta_1\varepsilon} - A_2(\gamma,\nu)e^{-\beta_1\varepsilon}e^{-\beta_1(n+1)}, \qquad (24b)$$

when $\gamma \le \varepsilon \le 1$

$$\Delta T[n,\varepsilon] = \frac{2\Delta T_{0m}}{\beta_1\gamma(1-D\delta)} A_2(\gamma,\nu)e^{-\beta_1\varepsilon}(1 - e^{-\beta_1(n+1)})]. \qquad (24c)$$

In using expressions (24) it is necessary to recall that in $A_1(\gamma, \nu)$, $A_2(\gamma, \nu)$, and $A_3(\gamma, \nu)$ instead of q_ν it is necessary to substitute its value for the case in question ($q_\nu = q_{1n} = -\beta_1$). Substituting $\varepsilon = \gamma$ and $\varepsilon = 0$ in (24b), (24a), we get the equations of the envelopes for the maximum and minimum values of the temperatures, respectively.

Relations (24) were used to construct the curves in Fig. 2b.

For an exponential pulse sequence the changes in thermistor temperature based on (15) and (21) will be ($k_i = 1$):
for $0 \le \varepsilon \le \gamma/2$

$$\Delta T[n, \varepsilon] = \frac{\beta_1 \Delta T_{0m}}{a(1-D\delta)}\left[\left(\frac{e^{-q_1\varepsilon}}{q_1-\beta_1} + \frac{1}{\beta_1}\right) + A_4(\gamma, \nu)e^{-\beta_1\varepsilon} - A_5(\gamma, \nu)e^{-\beta_1\varepsilon}e^{-\beta_1(n+1)}\right], \quad (25a)$$

for $\gamma/2 \le \varepsilon \le \gamma$

$$\Delta T[n, \varepsilon] = \frac{\beta_1 \Delta T_{0m}}{a(1-D\delta)}\left[a\left(\frac{e^{-q_1\gamma/2}}{\beta_1} - \frac{e^{q_1(\gamma/2-\varepsilon)}}{q_1-\beta_1}\right) + \frac{2a}{\gamma\beta_1^2}(1-\beta_1\varepsilon)e^{-q_1\gamma/2} + A_6(\gamma, \nu)e^{-\beta_1\varepsilon} - A_5(\gamma, \nu)e^{-\beta_1\varepsilon}e^{-\beta_1(n+1)}\right], \quad (25b)$$

for $\gamma \le \varepsilon \le 1$

$$\Delta T[n, \varepsilon] = \frac{\beta_1 \Delta T_{0m}}{a(1-D\delta)} A_5(\gamma, \nu)[1 - e^{-\beta_1(n+1)}]e^{-\beta_1\varepsilon}. \quad (25c)$$

Since the maximum value of ΔT occurs at $\varepsilon = \gamma$, and the minimum value at $\varepsilon = 0$, the equations of the envelopes can be determined on the basis of (25b) and (25a).

The curves in Fig. 2c are based on relations (25).

Pulsed variation of the dissipation factor. The dissipation factor is proportional to the heat transfer coefficient [6]:

$$k = \alpha F.$$

Thus, a change in dissipation factor is analogous to a change in heat transfer conditions.

For the given case we shall take U_c = const, R = = const, T_0 = const. For k = var expression (16) takes the form

$$\Delta P_\alpha = k_1 \Delta T - (T_1 - T_0)\Delta k. \quad (26)$$

After substitution of (18), (19a), and (26) in (19b) and certain transformations, we get the transfer function of the circuit containing the thermistor:

$$K_n(p) = \frac{\Delta T(p)}{\Delta k(p)} = \frac{T_1 - T_0}{k_1(1-D\delta)} \frac{1}{(\tau_e p + 1)} = \frac{P_n(p)}{Q_n(p)}. \quad (27)$$

With account for (3), Eq. (27) becomes

$$K_n(q) = \frac{T_1 - T_0}{k_1(1-D\delta)} \frac{\beta_1}{(q+\beta_1)} = \frac{P_n(q)}{Q_n(q)},$$

where $\beta_1 = \tau/\tau_e$. The given function has one root q_{1n} = $= -\beta_1$. Using (6), (10), and (14), we get

$$c_{00} = \frac{(T_1 - T_0)}{k_1} \frac{k_i}{(1-D\delta)}, \quad (28)$$

$$c_{10} = -\frac{(T_1 - T_0)}{k_1} \frac{k_i}{(1-D\delta)},$$

$$c'_{10} = \frac{(T_1 - T_0)}{k_1 \tau} \frac{k_i \beta_1}{(1-D\delta)}, \quad (29)$$

$$c_{101} = \frac{(T_1 - T_0)}{k_1 \beta_1} \frac{k_i}{(1-D\delta)}. \quad (30)$$

Using (9), (13), (15), and (28)–(30), we get expressions for the changes of temperature for the various pulse sequences. As before, $k_i = 1$ (i.e., $x_0 = A_m =$ $= \Delta k_m$). For square pulses:
when $0 \le \varepsilon \le \gamma$

$$\Delta T[n, \varepsilon] = \frac{T_1 - T_0}{k_1} \frac{\Delta k_m}{(1-D\delta)} \times \left[1 - \frac{1-e^{-\beta_1(1-\gamma)}}{1-e^{-\beta_1}} e^{-\beta_1\varepsilon} + \frac{1-e^{\beta_1\gamma}}{1-e^{-\beta_1}} e^{-\beta_1(n+1+\varepsilon)}\right],$$

when $\gamma \le \varepsilon \le 1$

$$\Delta T[n, \varepsilon] = \frac{T_1 - T_0}{k_1} \frac{\Delta k_m}{(1-D\delta)} \times \left[\frac{1-e^{-\beta_1\gamma}}{1-e^{-\beta_1}} e^{-\beta_1(\varepsilon-\gamma)} + \frac{1-e^{\beta_1\gamma}}{1-e^{-\beta_1}} e^{-\beta_1(n+1+\varepsilon)}\right].$$

For triangular pulses:
when $0 \le \varepsilon \le \gamma/2$

$$\Delta T[n, \varepsilon] = \frac{2}{\gamma\beta_1} \frac{T_1 - T_0}{k_1} \frac{\Delta k_m}{(1-D\delta)} [(\beta_1\varepsilon - 1) + A_1(\gamma, \nu)e^{-\beta_1\varepsilon} - A_2(\gamma, \nu)e^{-\beta_1\varepsilon}e^{-\beta_1(n+1)}],$$

when $\gamma/2 \le \varepsilon \le \gamma$

$$\Delta T[n, \varepsilon] = \frac{2}{\gamma\beta_1} \frac{T_1 - T_0}{k_1} \frac{\Delta k_m}{(1-D\delta)} [1 + \beta_1(\gamma - \varepsilon) + A_3(\gamma, \nu)e^{-\beta_1\varepsilon} - A_2(\gamma, \nu)e^{-\beta_1\varepsilon}e^{-\beta_1(n+1)}],$$

when $\gamma \le \varepsilon \le 1$

$$\Delta T[n, \varepsilon] = \frac{2}{\gamma\beta_1} \frac{T_1 - T_0}{k_1} \frac{\Delta k_m}{(1-D\delta)} A_2(\gamma, \nu)e^{-\beta_1\varepsilon}[1 - e^{-\beta_1(n+1)}].$$

For exponential pulses:
when $0 \le \varepsilon \le \gamma/2$

$$\Delta T[n, \varepsilon] = \frac{\beta_1}{a} \frac{T_1 - T_0}{k_1} \frac{\Delta k_m}{(1-D\delta)} \left[\left(\frac{e^{-q_1\varepsilon}}{q_1-\beta_1} + \frac{1}{\beta_1}\right) + A_4(\gamma, \nu)e^{-\beta_1\varepsilon} - A_5(\gamma, \nu)e^{-\beta_1\varepsilon}e^{-\beta_1(n+1)}\right],$$

when $\gamma/2 \le \varepsilon \le \gamma$

$$\Delta T[n, \varepsilon] = \frac{\beta_1}{a} \frac{T_1 - T_0}{k_1} \frac{\Delta k_m}{(1 - D\delta)} \times$$
$$\times \left[a \left(\frac{e^{q_1\gamma/2}}{\beta_1} - \frac{e^{-q_1(\gamma/2-\varepsilon)}}{q_1 - \beta_1} \right) + \frac{2a}{\gamma_1 \beta_1^2} (1 - \beta_1 \varepsilon) e^{-q_1\gamma/2} + \right.$$
$$\left. + A_6(\gamma, \nu) e^{-\beta_1 \varepsilon} - A_5(\gamma, \nu) e^{-\beta_1 \varepsilon} e^{-\beta_1(n+1)} \right],$$

when $\gamma \leq \varepsilon \leq 1$

$$\Delta T[n, \varepsilon] =$$
$$= \frac{\beta_1}{a} \frac{T_1 - T_0}{k_1} \frac{\Delta k_m}{(1 - D\delta)} A_5(\gamma, \nu)[1 - e^{-\beta_1(n+1)}] e^{-\beta_1 \varepsilon}.$$

If we take the ratio of changes of temperature due to the action of pulses of ambient temperature and dissipation factor as equal to unity, then, irrespective of the shape of the pulse, we shall have the following relation between the pulse amplitudes (provided that γ, β, a, D and δ are fixed):

$$\Delta T_{0m} = \frac{T_1 - T_0}{k_1} \Delta k_m. \tag{31}$$

It follows from (31) that for the same amplitude the action on the R $-$ R_T circuit of a sequence of pulses of dissipation factor will cause a $(T_1 - T_0)/k_1$ times greater change in thermistor temperature than the action of a sequence of ambient temperature pulses.

Using the equivalent circuit [5], it is easy to determine the transfer function for change in circuit voltage and apply all that has been said to the given case.

In conclusion, we note that the expressions obtained can be used to calculate circuits with thermistors for stepwise, sawtoothed (Fig. 1) and continuously growing (both linearly and exponentially) actions of the above-examined parameters. For this it is sufficient to take $\gamma = 1$, and, in addition, for continuously growing input variables n = 0.

NOTATION

A_m – pulse amplitude; τ – pulse repetition period; t_i – pulse length; k_i – amplification factor; ε – real parameter (time); ν – number of root of equation; $s(\varepsilon)$ – pulse shape; n – number of pulse; P_α – dissipated power; k – dissipation factor; T_0 – ambient temperature; U_c – circuit voltage; α_T – temperature coefficient of thermistor resistance; R_T – thermistor resistance; R – linear resistance; U_T – voltage drop across thermistor; C_V – volume heat capacity of thermistor; P_T – power supplied to thermistor; $\tau_e = \tau_\theta/(1 - D\delta)$ – time constant of electrical circuit; τ_θ – thermistor thermal time constant; $D = -\alpha_T(P_\alpha/k) = (B/T^2)(T - T_0)$ – relative power sensitivity of thermistor; $\delta = (R_T - R)/(R_T + R)$ – dimensionless circuit parameter characterizing power supply regime; α – heat transfer coefficient; F – surface area of thermistor; T – temperature of thermistor. The subscript m indicates the maximum of the corresponding quantity, the subscript 1 on k and T denotes values preceding the start of the transient process.

REFERENCES

1. Ya. Z. Tsypkin, Theory of Linear Pulse Systems [in Russian], Fizmatgiz, 1963.

2. M. F. Gardner and J. L. Barnes, Transients in Linear Systems [Russian translation], Fizmatgiz, 1961.

3. G. K. Nechaev, Thermistors in Automatic Control [in Russian], Gostekhizdat, Kiev, 1962.

4. A. G. Shashkov and A. S. Kasperovich, Dynamic Properties of Circuits Containing Thermistors [in Russian], GEI, 1962.

5. V. N. Stanishevskii and V. A. Palagin, Izv. vuzov, Energetika, no. 9, 1965.

6. A. V. Luikov, Theory of Heat Conduction [in Russian], Gostekhizdat, 1952.

5 July 1965
Institute of Heat and Mass Transfer, AS BSSR, Minsk